

# Theory of phase fluctuating d-wave superconductors and the spin response in underdoped cuprates

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The minimal theory of spin of gapless quasiparticles coupled to fluctuating vortex defects in the phase of the d-wave superconducting order parameter at  $T = 0$  is studied. We find a single superconductor-spin density wave phase transition, which may be fluctuation induced first-order, at which vortices condense and the chiral symmetry for fermions dynamically breaks. We compute the spin-spin correlation function in the fluctuating superconducting state, and discuss some prominent trends in the neutron scattering data on underdoped cuprates in light of our results.

The underdoped high temperature superconductors are highly anisotropic, quasi two-dimensional materials, in which thermal and quantum fluctuations of the phase of the superconducting order parameter (OP) should be important well below the pseudogap temperature [1]. It has recently been shown that a phase fluctuating d-wave superconductor (dSC) at zero temperature ( $T = 0$ ) is inherently unstable towards the formation of the insulating spin density wave (SDW) state with the loss of phase coherence [2], [3]. This result follows from the realization that the effective low-energy theory for the gapless d-wave quasiparticles, besides the usual spatial symmetries, also possesses an additional internal ("chiral") symmetry. This symmetry becomes dynamically broken when the superconducting phase coherence is lost via proliferation of vortex defects. The effective theory of the phase fluctuating dSC in this formulations is closely related to the three dimensional quantum electrodynamics ( $QED_3$ ) [4], in which the "charge" is proportional to the vortex condensate [2], [4]. In first approximation the fluctuations of the gauge-field may be completely neglected in the superconducting state, which yields quasiparticles as well defined excitations. When the vortices condense and the charge in the effective  $QED_3$  becomes finite, the chiral symmetry of the dSC becomes dynamically broken [5]. In the present context this translates into the SDW ordering, with confined spin-1/2 (spinon) excitations [2].

Several issues of direct relevance for underdoped cuprates naturally arise in such a theory of the phase fluctuating dSC. Can the SC and the SDW long-range orders coexist? Several intriguing recent experiments [6], [7], [8], find possible signs of such coexistence of the two inimical orderings. If there is a single dSC-SDW quantum phase transition, on the other hand, what should be its universality class? What are the effects of the vortex fluctuations *inside* the superconducting state? These questions all call for a better understanding of the quasiparticle-vortex interaction, particularly in the superconducting state. In this Letter we investigate probably the simplest theory of the low-energy d-wave quasiparticles coupled to the complex scalar vortex field (dual to the superconducting OP). The theory describes the decoupled spin sector of the phase coherent dSC at  $T = 0$  and subject to strong quantum vortex fluctuations. In underdoped high temperature superconductors such fluctua-

tions should presumably arise from the Coulomb interaction that becomes effectively stronger towards half filling, and tends to disorder the phase of the superconducting OP. If extended to the non-superconducting phase, our theory reduces to the previously studied  $QED_3$ . First, we argue that the condensation of vortices (i. e. the loss of superconductivity) and the breaking of the chiral symmetry for fermions (the SDW instability) in our theory coincide. Second, we show that the quantum dSC-SDW phase transition can be either in the modified XY universality class, or it could be fluctuation-induced first-order, depending on the values of the couplings in the theory. Finally and most importantly, we calculate the spin dynamics *induced* by vortex fluctuations deep inside the dSC in the leading approximation. The intricate evolution of the spin response with frequency observed in inelastic neutron scattering experiments on YBCO [9],[10], [11] follows quite naturally from our results. In particular, we predict the appearance of four weak and narrow "diagonally" incommensurate peaks at low energies, the energy of which vanishes with the superconducting  $T_c$ .

Consider the quantum mechanical ( $T = 0$ ) action for the low energy quasiparticles in the two-dimensional phase fluctuating dSC,  $S = \int d^3x \mathcal{L}$ ,  $x = (\tau, \vec{r})$ , and

$$\mathcal{L} = \sum_{i=1}^N \bar{\Psi}_i \gamma_\mu (\partial_\mu - i a_\mu) \Psi_i + \frac{i}{\pi} \vec{a} \cdot (\nabla \times \vec{A}) + \quad (1)$$

$$|(\nabla - i \vec{A})\Phi|^2 + \mu^2 |\Phi|^2 + \frac{b}{2} |\Phi|^4.$$

Here the fluctuating complex field  $\Phi$  describes the vortex loops, and  $\langle \Phi \rangle = 0$  implies the superconducting phase coherence. Two ( $N = 2$ ) four-component Dirac fermions describe the gapless, neutral, spin-1/2 (spinon) excitations near the four nodes of the superconducting order parameter, one Dirac field for each pair of diagonally opposed nodes [2].  $\vec{a}$  is the gauge-field that results from properly absorbing the singular part of the superconducting phase due to vortices into the spinon fields [4], [2].  $\gamma_\mu$ ,  $\mu = 0, 1, 2$  are the Dirac gamma matrices [2]. Finally,  $\vec{A}$  is an additional auxiliary (Chern-Simons) field which facilitates the statistical spinon-vortex coupling, as will be explained shortly. The single tuning parameter  $\mu^2$  may be understood as related to doping,  $\mu^2 \propto (x - x_c)$ , with  $x_c$  being the critical doping in the

underdoped regime. The coupling  $b$  describes the short-range repulsion between the vortex loops. In our units  $\hbar = c = v_F = v_\Delta = k_B = e = 1$ .

We may motivate the field theory (1) as follows [12]: the integration over the gauge-field  $\vec{a}$  leaves the spinon-vortex coupling  $i\vec{A} \cdot \vec{J}_\Phi$ , where  $\nabla \times \vec{A} = \pi \vec{J}_\Psi$ , and  $\vec{J}_\Psi$  and  $\vec{J}_\Phi$  are the spinon and vortex current densities, respectively. Vortices and spinons therefore see each other as sources of magnetic flux, and circling around a vortex with a spinon (or vice versa) leads to a phase change of  $\pi$ . This is precisely the *statistical* (Aharonov-Bohm) part of the interaction between vortices and spinons [4], [2], [12]. Without fermions ( $N = 0$ ), the integration over the gauge fields leads to the standard  $|\Phi|^4$  theory, in the universality class of the XY model. Strictly speaking, for  $N = 0$  the theory should reduce to the Higgs scalar electrodynamics, dual to the  $|\Phi|^4$  theory. This can be accomplished by introducing an additional fluctuating gauge field into the theory that would mediate a long-range interaction between vortices and represent the electrical charge. Following the previous work [2], [4], [13] we will assume spin and charge to be separated at low energies, so we may neglect the charge degrees of freedom. This assumption is surely justified inside the superconducting state which is the subject of the present work. For subtle issues that arise in the normal state the reader is referred to [14]. Finally, the velocity anisotropy ( $v_F \gg v_\Delta$ ) and all the local interactions between quasiparticles [2] have been neglected, as irrelevant at low energies [15], [16].

Let us discuss first how the theory (1) is related to the  $QED_3$  [2], [4], in the simplest mean-field approximation for the vortex field. Neglecting the fluctuations in  $\Phi$ , in the superconducting phase ( $\langle \Phi \rangle = 0$ ), the integration over  $\vec{A}$  constrains  $\nabla \times \vec{a} = 0$ , and spinons are essentially free. When vortices condense ( $\langle \Phi \rangle \neq 0$ ), on the other hand, the gauge-field  $\vec{A}$  acquires a mass via Higgs mechanism,  $|\langle \Phi \rangle|^2 \vec{A}^2$ . The Gaussian integration over  $\vec{A}$  produces then the Maxwell term for  $\vec{a}$ ,  $\sim (\nabla \times \vec{a})^2 / |\langle \Phi \rangle|^2$ . Together with the Dirac Lagrangian this constitutes the  $QED_3$  for spinons, in which the fermions' chiral symmetry generated by  $\gamma_3$  and  $\gamma_5$  is dynamically broken by the generation of the mass term  $\sim M \bar{\Psi}_i \Psi_i$  [5]. In the present context such a mass  $M \sim \langle \bar{\Psi}_i \Psi_i \rangle \sim |\langle \Phi \rangle|^2$  is proportional to the SDW OP, with the (incommensurate) ordering wave vectors that connect the diagonally opposed nodes of the superconducting OP [2]. Carefully including the charge degrees of freedom in (1) one also finds the SDW to be an electrical insulator [14].

In the mean-field approximation, the vortex condensation and the SDW transition in the theory (1) coincide. It is not obvious that this feature survives the inclusion of the fluctuations, and it is conceivable that the SDW transition may occur within the superconducting phase [2], [17]. To examine this issue we first ask what would be the *exact* propagator for the gauge field  $\vec{a}$  if there were no fermions, i. e. when  $N = 0$  in (1). We then approximate the interacting action for  $\vec{a}$  that would result from

the integrations over  $\Phi$  and  $\vec{A}$  with the effective Gaussian term that reproduces that exact propagator. In the Landau gauge such a propagator is

$$G_{aa,\mu\nu}^0(p, m) = \frac{\pi^2 \Pi_{AA}(p, m)}{p^2} (\delta_{\mu\nu} - \hat{p}_\mu \hat{p}_\nu), \quad (2)$$

where  $\Pi_{AA}(p, m)(\delta_{\mu\nu} - \hat{p}_\mu \hat{p}_\nu)$  is the transverse current-current correlation function in the  $|\Phi|^4$  theory. Here  $m^2 = \mu^2 + O(b)$  is the fully renormalized "mass" of the vortex field. In general,  $\Pi_{AA}(p, m) = npF_+(m/p)$ , for  $m^2 > 0$  (superconductor). To the lowest order in  $b$

$$F_+(z) = \frac{1}{8\pi} [(4z^2 + 1) \arctan(\frac{1}{2z}) - 2z] + O(b). \quad (3)$$

Here we generalized our model to the one with  $n$  complex vortex fields,  $n = 1$  being the case of physical interest.

We are now in position to study the chiral symmetry breaking for fermions with (2) serving as the bare (without fermion polarization) gauge-field propagator in the  $QED_3$ . Consider the standard large- $N$  Dyson equation [5] for the fermion self-energy

$$\Sigma(q) = \frac{1}{4} Tr \int \frac{d^3 p}{(2\pi)^3} \gamma_\mu \frac{G_{aa,\mu\nu}(p - q, m) \Sigma(p)}{p^2 + \Sigma(p)^2} \gamma_\nu \quad (4)$$

Here  $[G_{aa,\mu\nu}(p, m)]^{-1} = \Pi_{aa,\mu\nu}^F(p) + [G_{aa,\mu\nu}^0(p, m)]^{-1}$ , with the one-loop fermion polarization  $\Pi_{aa,\mu\nu}^F(p) = (Np/8)(\delta_{\mu\nu} - \hat{p}_\mu \hat{p}_\nu)$ . It is useful to consider first the point of the superconducting phase transition  $m = 0$ , where Eq. (3) implies  $\Pi_{AA}(p, 0) = np(1/16 + O(b_c))$ , where  $b_c$  is the fixed point value of  $b$  in  $d = 3$  [18]. Linear dependence of  $\Pi_{AA}(p, 0)$  on momentum is an exact result [19]. Inserting this into the Dyson equation we find that at  $m = 0$  the effect of vortices is only to increase the coefficient  $N \rightarrow N_{eff} = N + 128/(\pi^2 n) + O(b_c)$ . Recalling that the non-trivial solution of the Dyson equation (4) exists only for  $N_{eff} < N_c = 32/\pi^2$  [5], for  $N = 2$  we find that chiral symmetry at  $m = 0$  would be already broken only for  $n > n_c = 10.44(1 + 32/(9\pi^2 n))$ , where we have also included the known  $O(b_c)$  correction to  $\Pi_{AA}(p, 0)$  in the large- $n$  approximation [18]. Since  $n = 1$  in the physical case we conclude that right at the superconducting critical point SDW order is most likely absent.

Little further thought shows that the above result implies that the SDW order is absent for all  $m^2 > 0$ . Indeed, this is to be expected, since  $G_{aa}^0(p, m) < G_{aa}^0(p, 0)$ , and  $\vec{a}$  is only stiffer in the superconducting phase. To prove this, assume that for  $m^2 > 0$  a non-trivial solution of the Eq. (4),  $\tilde{\Sigma}(q)$ , does exist. Such a solution would then satisfy

$$\tilde{\Sigma}(q) < \frac{1}{4} Tr \int \frac{d^3 p}{(2\pi)^2} \gamma_\mu \frac{G_{aa,\mu\nu}(p - q, 0) \tilde{\Sigma}(p)}{p^2 + \tilde{\Sigma}(p)^2} \gamma_\nu. \quad (5)$$

On the other hand, we already established that there is only a trivial solution of the Dyson equation at  $m = 0$ . This means that assuming a candidate function  $\Sigma(p)$  and

inserting it under the integral in the Dyson equation (4) for  $m = 0$  will produce only a *smaller* new  $\Sigma(q)$ , since under iterations the physically acceptable self-energy must approach the trivial solution. This further implies that any  $\Sigma(q)$  from the domain of attraction of the trivial solution at  $m = 0$  has to satisfy the inequality opposite to (5). A non-trivial  $\tilde{\Sigma}(q)$  at  $m^2 > 0$  therefore can not exist if it did not exist at  $m^2 = 0$ .

For  $m^2 < 0$  superconductivity is lost, and  $\Pi_{AA}(p, m) = npF_- (|\langle\Phi\rangle|^2/p)$ , with  $F_-(z) \rightarrow 1/16$  for  $z \ll 1$ , and  $F_-(z) = 2z$ , for  $z \gg 1$  (Higgs mechanism), to the leading order. Since  $G_{aa}^0(p, m) > G_{aa}^0(p, 0)$  for  $m^2 < 0$ , the above argument no longer applies. In fact, it is easy to see that there is *immediately* a non-trivial solution of the Eq. (4) when  $m^2 < 0$ . Since  $\Sigma(q) > 0$  only for  $q \sim |\langle\Phi\rangle|^2$  and smaller,  $|\langle\Phi\rangle|^2$  serves as the effective ultraviolet cut-off in the Eq. (4). The Dyson equation then reduces to the standard one in the  $QED_3$  [5]. For  $N = 2 < N_c$ ,  $\Sigma(q = 0) \sim |\langle\Phi\rangle|^2$ , and the chiral symmetry is dynamically broken. There is no intermediate (quantum disordered) phase in between the SDW and the dSC in the theory (1), unless the exact value of  $N_c$  in the  $QED_3$  is actually less than two [20].

The above argument strongly suggests that there is a single dSC-SDW transition in the theory (1), but does not say anything about its nature. To study this issue it is better to proceed in the opposite direction, and integrate over the fermions first. This gives dynamics to  $\vec{a}$  via the fermion polarization bubble. If we neglect the quartic and the higher order terms and perform the Gaussian integral over  $\vec{a}$ , the result is the Maxwell-like term for  $\vec{A}$ :  $\langle A_\mu(p) A_\nu(-p) \rangle = (N\pi^2/8|p|)(\delta_{\mu\nu} - \hat{p}_\mu \hat{p}_\nu)$ . Note that since the inverse of the above average is non-analytic at  $p = 0$ , it can not renormalize [21], and therefore the number of fermions  $N$  represents an exactly *marginal* coupling. Assuming a constant  $\Phi$  in (1) we may further integrate  $\vec{A}$  to find the energy per unit volume to be

$$S[\Phi] - S[0] = \left(\frac{\mu^2}{\Lambda^2} + \frac{N}{48}\right)\Phi^2 + \frac{1}{2}\left(\frac{b}{\Lambda} - \frac{\pi^2 N^2}{48}\right)\Phi^4 + \frac{1}{6\pi^2}\ln\left(1 + \frac{\pi^2 N}{4}\Phi^2\right) + \frac{\pi^4 N^3}{384}\Phi^6 \ln\left(1 + \frac{4}{\pi^2 N\Phi^2}\right), \quad (6)$$

where we have rescaled  $\Phi^2/\Lambda \rightarrow \Phi^2$ , with  $\Lambda$  being the ultraviolet cutoff. Standard analysis of the Eq. (6) shows that there is a discontinuous transition for  $N > (4/\pi)\sqrt{2b/\Lambda}$ . Using the lowest order fixed point value  $b/\Lambda = (2\pi^2/5)(4-d) + O((4-d)^2)$  in  $d = 3$  as a crude estimate of this bound, we find the first-order transition for  $N > 3.58$ , in rough agreement with [12]. Of course, this conclusion is to be trusted only for  $N \gg 1$ , when the first-order transition occurs at large  $\mu^2$ , at which our neglect of fluctuations in  $\Phi$  in arriving at the Eq. (6) becomes justified. For  $N \ll 1$ , on the other hand, the phase transition in the theory (1) should be continuous, with weakly modified XY exponents:  $\nu = \nu_{xy} + O(N)$ ,  $\eta = \eta_{xy} + O(N)$  [12]. The situation is reminiscent of the Ginzburg-Landau superconductor [19], with  $N \gg 1$  anal-

ogous to the strongly type-I, and  $N \ll 1$  to the extreme type-II case.

Finally, we turn to spin dynamics deep inside the dSC, when  $m/b \gg 1$ . For small momenta,  $p \ll m$ , we find then that  $G_{aa,\mu\nu}^0(p, m) = ((\pi/24m) + O(p^2))(\delta_{\mu\nu} - \hat{p}_\mu \hat{p}_\nu)$ . Integrating out such a *massive*  $\vec{a}$  leads to the effective 2+1 dimensional Thirring model for fermions in the dSC

$$\mathcal{L} = \bar{\Psi}_i \gamma_\mu \partial_\mu \Psi_i + \frac{\pi}{72m} \bar{\Psi}_i \gamma_\mu \Psi_i \bar{\Psi}_j \gamma_\mu \Psi_j, \quad (7)$$

with the summation over repeated indices assumed. Using the Hubbard-Stratonovich transformation we can rewrite this as

$$\mathcal{L} = \bar{\Psi}_i \gamma_\mu \partial_\mu \Psi_i + \frac{18m}{\pi} Tr[M_\mu^{ij} M_\mu^{ji}] + Tr[M_\mu^{ij} \gamma_\mu \Psi_j \bar{\Psi}_i] \quad (8)$$

Making an *ansatz*  $M_\mu^{ij}(x) = (M(x)/3)\delta_{ij}\gamma_\mu$  at the saddle point, and then integrating out the fermions yields

$$S = N \int d^3x \left[ \frac{24m}{\pi} M^2(x) - Tr[\ln(\gamma_\mu \partial_\mu - M(x))] \right]. \quad (9)$$

Expanding further in powers of  $M(x)$  we finally write

$$\frac{S}{N} = \int \frac{d^3q}{(2\pi)^3} \left( \frac{24\pi m - \Lambda}{\pi^2} + \frac{|q|}{8} \right) M^2(q) + O(M^4), \quad (10)$$

where  $M^2(q) = M(q)M(-q)$ . On the other hand, using the definition of the Dirac fields in terms of the electron creation and annihilation operators [2]

$$\langle M(x) \rangle = \frac{\pi}{12Nm} \langle \hat{S}_z(\vec{r}, \tau) \rangle \sum_{i=1}^2 \cos(2\vec{K}_i \cdot \vec{r}), \quad (11)$$

where the vectors  $\pm\vec{K}_i$ ,  $i = 1, 2$  denote the positions of the four nodes of the d-wave order parameter.  $\langle M(q = 0) \rangle$  is therefore the static SDW OP [2]. Similarly,

$$\langle M^2(\vec{q}, \omega) \rangle - \frac{\pi}{48Nm} = (12)$$

$$\left( \frac{\pi}{24Nm} \right)^2 \sum_{i=1}^2 \left( \langle \hat{S}_z(\vec{q} + 2\vec{K}_i, \omega) \hat{S}_z(-\vec{q} - 2\vec{K}_i, -\omega) \rangle + \langle \hat{S}_z(\vec{q} - 2\vec{K}_i, \omega) \hat{S}_z(-\vec{q} + 2\vec{K}_i, -\omega) \rangle \right).$$

Since the dSC is rotationally invariant, the spin-spin correlation function is diagonal,  $\langle \hat{S}_\alpha(\vec{k}, \omega) \hat{S}_\beta(-\vec{k}, -\omega) \rangle = \chi(\vec{k}, \omega) \delta_{\alpha\beta}$ . We may therefore finally deduce that the imaginary part of the spin response function  $\chi(\vec{k}, \omega) = \chi'(\vec{k}, \omega) + i\chi''(\vec{k}, \omega)$  in the phase fluctuating dSC is

$$\chi''(2\vec{K}_i \pm \vec{q}, \omega) = \left( \frac{12Nm}{\pi} \right)^2 Im \langle M^2(\vec{q}, \omega) \rangle. \quad (13)$$

Analytically continuing to *real* frequencies  $i\omega \rightarrow \omega$ , in the Gaussian approximation to Eq. (10) we finally obtain

$$\chi''(2\vec{K}_i \pm \vec{q}, \omega) = \left( \frac{24m}{\pi} \right)^2 \frac{N\Theta(\omega^2 - q^2)\sqrt{\omega^2 - q^2}}{(192m/\pi)^2 + \omega^2 - q^2}, \quad (14)$$

where we have taken  $m \gg \Lambda$  deep inside the dSC. With  $m \rightarrow \infty$  Eq. (14) reduces correctly to the non-interacting limit, in agreement with [22].

First, let us connect the uniform vortex susceptibility  $m$  to the measurable superconducting  $T_c$ . Assuming a continuous dSC-SDW transition,  $m^2 \propto (\mu^2 - \mu_c^2)^\gamma$ , with  $\gamma = \nu(2 - \eta)$  characterizing the quantum critical point; also,  $T_c \propto (\mu^2 - \mu_c^2)^{z\nu}$ . Since the dynamical critical exponent in the theory (1) is  $z = 1$ ,  $m \propto T_c^{1-(\eta/2)}$ . Sufficiently away from the critical point we may neglect the anomalous dimension  $\eta$  and find the result dictated by the (engineering) dimensional analysis,  $192m/\pi = cT_c$ , where  $c$  is a number.

Next, we look for the position of the maximum of the spin response function  $\chi''(\vec{q}, \omega)$  at a fixed energy  $\omega$ . Eq. (14) implies that for  $\omega \ll cT_c$ , the maximum values are located at four "diagonally" *incommensurate* wave vectors  $\pm 2\vec{K}_{1,2}$  (i. e. at  $\vec{q} = 0$ ). As frequency increases the peak intensity grows, and recalling that  $v_F \gg v_\Delta$  and taking the lattice periodicity into account one finds the four peaks overlapping first at four "parallel" *incommensurate* positions [22]. With the further increase of frequency, at some point four initial peaks start overlapping at  $(\pi, \pi)$ , and for a while the "commensurate" response dominates. At  $\omega > cT_c$  the maximum in Eq. (14) shifts to a  $|\vec{q}| \neq 0$ , which implies a weak redistribution of the commensurate peak to four "parallel" *incommensurate* positions at largest frequencies. The predicted evolution of the spin response is in qualitative agreement with the observations in YBCO [11]. More detailed and quantitative

picture will be presented in a future publication.

Consider then the commensurate response at  $\vec{k} = (\pi, \pi)$ , that in our notation corresponds to some  $\vec{q} \neq 0$ . As a function of frequency, at  $T = 0$   $\chi''$  vanishes below some cutoff energy  $\omega_c = |\vec{q}|$ . Also, the maximum response is at the energy  $\omega_0 = \sqrt{|\vec{q}|^2 + (cT_c)^2}$ , which *decreases* with decreasing  $T_c$ , as observed [9], [10]. As  $T_c \rightarrow 0$ , however, the commensurate peak energy  $\omega_0 \rightarrow |\vec{q}| \neq 0$ . Such a finite  $\omega_0$  ( $\approx 20\text{meV}$ ) as  $T_c \rightarrow 0$  is in agreement with the data in YBCO (Fig. 29 in the second reference [9]). Remarkably, the cutoff energy that can be estimated from a different set of data on YBCO (Fig. 17 in [10]) is also  $\omega_c \approx 20\text{meV}$ , consistent with our prediction. At large energies,  $\chi''$  should behave as  $\sim 1/\omega$ , which also appears to be in general accord with the data.

Finally, at the incommensurate ( $\vec{q} = 0$ ) wavevector,  $\chi''(2\vec{K}_i, \omega) \sim \omega$  at small energies, with the peak at  $\omega_0 = cT_c$ . Whereas the energy of the resonance should go to a finite value as  $T_c$  vanishes with underdoping, we predict that the energy of the "diagonally" *incommensurate* peaks, which should become discernible at lower energies, should extrapolate to zero. Detection of such a soft mode inside the dSC state would provide the smoking gun evidence for (1) as the low-energy theory for spin of underdoped cuprates.

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